

UDC 004

Geogebra: Calculation of Centroid¹ Qamil Kllogjeri² Pellumb Kllogjeri

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Abstract. Our paper is result of the research done in a special direction for solving problems of physics by using GeoGebra programme: calculation of centroid. Lots of simulations of physical phenomena from the class of Mechanics can be performed and computational problems can be solved with GeoGebra. GeoGebra offers many commands and one of them is the command "centroid" to calculate the coordinates of the centroid of a polygon but, we have created a new tool to calculate the coordinates of the centroid of a plane region bounded by curves. Our work is part of the passionate work of many GeoGebra users which will result with a very rich fund of GeoGebra virtual tools, examples and experiences that will be worldwide available for many teachers and practioners.

Keywords: centroid; tool of centroid; advantages of GeoGebra.

Introduction. GeoGebra is a dynamic mathematics open source software for learning and teaching mathematics in schools. It was developed by Markus Hohenwarter and an international team of programmers. They have done a brilliant work and from the time that GeoGebra was introduced until now are done a great number of progressive steps by many mathematics teachers and lecturers throughout the world. GeoGebra combines geometry, algebra, statistics and calculus. Geogebra, provides the basic features of Computer Algebra System to bridge gaps between Geometry, Algebra and Calculus. The software links the geometric constructions shown in Geometry window to the analytic equations and coordinates representations and graphs shown in Algebra window.

The default GeoGebra view is consisted of the Algebra View, the Graphics View, the InputBar, and the Tool Bar. There are two ways to construct an object in GeoGebra: use of the Tools in the Toolbar, or use of the corresponding Command entered in the Input Bar. Regardless of how is constructed an object, the algebraic representation of the object will appear in the Algebra View. It is this dual representation of objects – visually in the Graphics View and Algebraically in the Algebra View – which makes GeoGebra so powerful.

We can do geometric constructions on the drawing pad of the graphics window and, on the other hand, we can directly enter algebraic input, commands, and functions into the Input field by using the keyboard. The geometric constructions are done by the mean of the main virtual tools, which are found in the set of the toolboxes which are to be opened, selected, activated and used during the construction process. In the toolboxes are found the virtual tools with their names linked with their functions like: New point, Move, Line through two points, Segment between two points etc., alongside which is their picture also. There are also buttons like: Delete object, Move drawing pad, Zoom in / Zoom out, Undo / Redo buttons etc... GeoGebra offers more commands than geometry tools [1].

GeoGebra environment is very pleasant and attractive because it has game features. The virtual tools of GeoGebra can be easily used and played with by anyone of the whole school system. According to Hohenwarter and Preiner (2007), GeoGebra appears to be a friendly software that can be operated intuitively and does not require advanced skills to get started [2].

Based on the reports and lessons given in different GeoGebra conferences and other GeoGebra resources is found that the use of GeoGebra fosters students' perseverance, curiosity, inductive attitudes, and inclination to seek accuracy and rigor in geometric learning tasks. Also, are found complex and effective interactions between computer technology, mathematics, and the classroom

environment. GeoGebra provides a very comfortable and fruitful environment for research work. This environment provides a higher quality education for students and the capable teachers who implement creative learning environments with technology for the purpose of maximizing the students' learning success that are desperately needed for today and the future (Adler et al., 2005, p. 360). [3]

GeoGebra is a new, cost-free, and very innovative technology that can be used to support the progressive development of mental models appropriate for solving complex problems involving mathematical relationships. Also, it is supported with many additional free resources, including lessons, examples, and activities that can be used to support the training of teachers in the integration of GeoGebra into curricula aligned with standards, goals and objectives. The range of the topics in which GeoGebra is used is very wide, including model real-world problems and support problem-solving, provisions of visualizations and interactive illustrations, and improvement of student motivation and cognitive development. Teachers and engineers must have interest in the integration of GeoGebra or similar technologies in mathematics education and technical problems solving.

Our work is part of the world-wide and ongoing efforts of GeoGebra and other technology communities done to increase the pace of change in development of new tools to facilitate and faster the doing of mathematics, the teaching and learning of mathematics, and teacher education itself. [4]. "...It is known that as soon as one innovation is engaged with, another becomes available – but we have little sense of whether all of this activity results in better mathematics classrooms" (Mously et al., 2003, p. 395) [5]. However, all scientific communities have to press on with their research work, the time will prove its correctness and value.

Through the online GeoGebra Wiki and global and local professional conferences, an international community of GeoGebra users is daily growing and is actively addressing traditional problems in mathematics education and developing new pedagogical and theoretical issues on mathematics teaching and learning, while taking advantage of both technological and theoretical inventions. We emphasize that using the computational tool of GeoGebra for teaching functions and their applications in the construction of mathematical models that explain certain physical phenomena is a learning experience with teachers and students where the application of GeoGebra is made to model the real problem in a mathematical problem. By working together, teachers and students and Geogebra users in general, can and will achieve a greater level of generalization and abstraction of the function concept than the one achieved with the traditional resources of pen and paper. They will, also, develop faster computational algorithms by the use of GeoGebra, reducing this way the efforts in the algebraic and geometric procedures. General tools for mathematics education include dynamic geometry software, computer algebra systems, spreadsheets, and dynamic mathematics software etc. Fuglestad defines such tools "as open and flexible software, not made for specific topics or limited to teach specific tasks. [. . .]. Such tools can be used for a wide variety of problems and can provide learning situations to explore and experiment with mathematical connections, and provide new ways of approaching the tasks" (Fuglestad, p.1) [6]. GeoGebra tool plays an important role in simulations of physical phenomena, allowing to move ideas into images that can be appreciated by our senses. This is intended to give meaning to an abstract understanding of these physical phenomena. GeoGebra is a valuable resource for all educators interested in promoting the development of critical reasoning skills, also for GeoGebra users. It can be downloaded for free and there are different versions of GeoGebra. The newest software is GeoGebra4 and GeoGebra Prim with new features added to the former ones. It can be downloaded from <http://www.geogebra.org> [7].

Supportive concepts and formulas. The concept of centroid of plane areas is similar to the centre of mass of **thin** uniform plates of the same shape as the plane areas. For a plane area, the centre of mass is commonly referred to as a **centroid**. The center of mass or centroid of a thin plate with uniform density ρ (or plane region) is the point in which the plate will be perfectly balanced horizontally if suspended from that point or, the moment of area about any axis through that point is zero. [8], [9], [10]. So, let's suppose that the plate is the region bounded by two curves $y = f(x)$ and $y = g(x)$, with variable values on the interval $[a, b]$. The coordinates of centroid are denoted and given by formulas as following:

Case 1: the plane region is bounded by the graph of $y = f(x)$ and **x-axis**,

$$\bar{x} = \frac{1}{A} \int_a^b x f(x) dx \text{ and } \bar{y} = \frac{1}{2A} \int_a^b f(x)^2 dx \text{ where,}$$

$A = \int_{\alpha}^{\beta} [f(x)]dx$ is the area of the region.

Case 2: the plane region is bounded by the graphs of two functions $y = f(x)$ and $y = g(x)$,

$$\bar{x} = \frac{1}{A} \int_{\alpha}^{\beta} x[f(x) - g(x)]dx \text{ and } \bar{y} = \frac{1}{2A} \int_{\alpha}^{\beta} [f(x)^2 - g(x)^2]dx$$

where, $A = \int_{\alpha}^{\beta} [f(x) - g(x)]dx$ is the area of the plane region between the curves.

GEOGEBRA USE IN PHYSICS. Our main goal is to explore GeoGebra software how can it be used to calculate the centroid of an enclosed plane region bounded by curves and x-axis or by two curves. So far, GeoGebra is used to find the centroid of a polygon by executing the command “centroid”. To use GeoGebra to calculate the curved regions are needed new tools.

Centroid of a polygon. GeoGebra can be used for solving problems of physics. One useful command is the one for calculating the center of mass: “centroid” command. This command is applicable for finding the centroid of any polygon that is well-studied in Physics.[11] So, constructing a polygon using the tool “polygon” we observe that it is displayed in algebra window as an equation: poly1 = <number>. The number here represents the area of the polygon. By applying “centroid “ command, in the *Input field* are taken the coordinates of the center of mass of the polygon, displayed in algebra window as an algebraic object, whereas the mass center point is shown in geometry window within or outside the polygon. It depends on its shape.

Centroid of a plane shape bounded by a curve and the horizontal axis

Consider the case of the enclosed region (or plate of such shape) between the second grade curve $f(x) = -1/4 x^2 + 9/4 x - 2$ and the horizontal axis (Fig.1). The intersection points of the curve with x-axis are $A = (1,0)$ and $B = (8,0)$. The existing GeoGebra tool for centroid cannot calculate the centroid of a region bounded by curves. The existing tool for centroid does function for polygons alone. We need to create a new tool to calculate the centroid of such shape. If we want to create a mathematical object (not included in the toolbar) many times, we may want to create our own tool. This will save us the trouble of creating similar mathematical objects repeatedly.

The new tool will function for the calculation of the centroids of the plane shapes bounded by x-axis and the graph of any second grade function with general form $f(x) = a x^2 + b x + c$. To create the new tool we apply the following procedure.

The first stage of the procedure: creating the sliders and graphing the function.

1. Open GeoGebra window and in the *View* menu select *Axes* and *Grid* options in order to display the coordinate *Axes*. We need them and this done by clicking one at a time **View>Axes>Grid** from the menu bar.

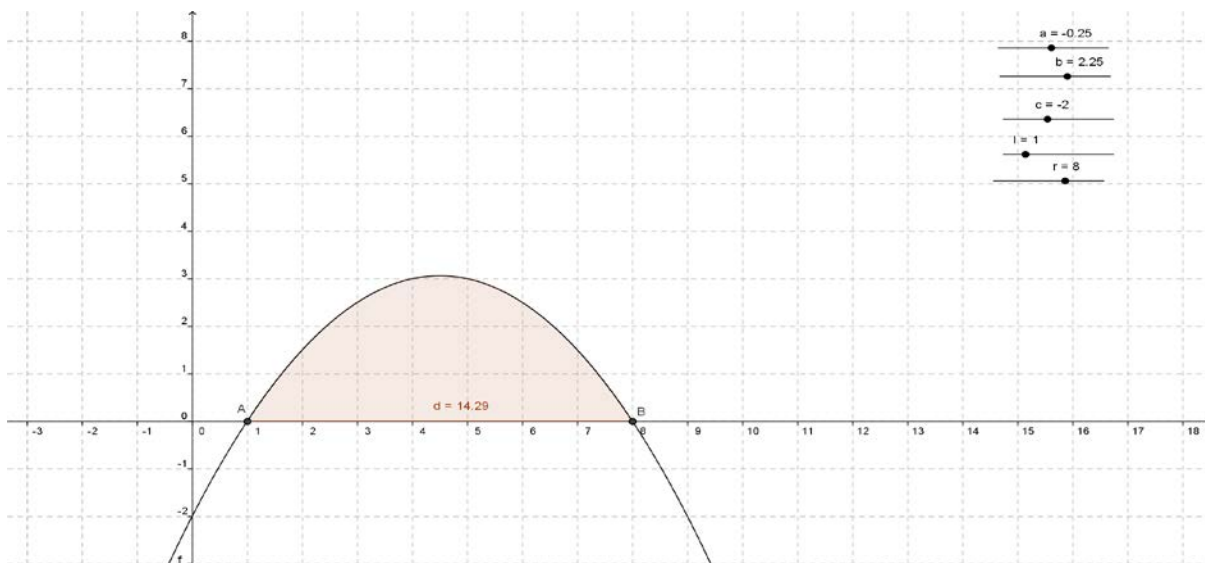
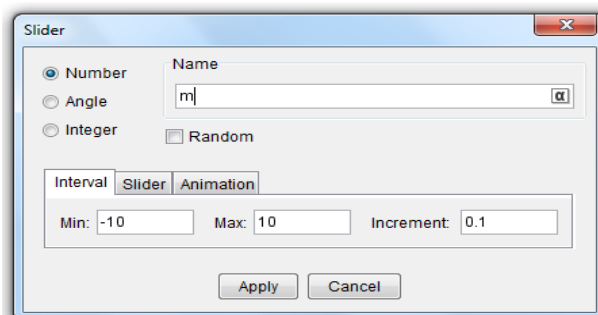


Fig. 1

2. Click on the tool box of the Slider Tool with icon and select it (In GeoGebra, a slider is the graphical representation of a free number or free angle).

3. Create a slider by showing this object in the Graphics View. This is done by clicking anywhere on the *Graphics view* to display the *Slider dialog box*. In the *Slider* dialog box, we see the slider's name a which can be changed to m , the interval of values of a , and the increment which can be changed as shown below or we can leave the values as they are.[12] The created slider named with a corresponds to the coefficient (parameter) a of the second grade function.



Finally, click the **Apply** button to finish the creation of slider a .

4. We create in the same way the sliders corresponding to coefficients b and c of the second grade function and the sliders l and r corresponding to the left intersection point of the curve with x -axis and to the right intersection point of the curve with x -axis, respectively. In general, the sliders l and r will correspond to the left bound and the right bound of the enclosed plane region, respectively.

5. Arrange the positions of the sliders by moving the sliders (firstly selecting the *Move* button) at the values: $a = -1/4$, $b = 9/4$, $c = -2$, $l = 1$ and $r = 8$. These are the values of the parameters of the second grade function and of the left and right bound of the plane region.

6. To graph the function $f(x) = -1/4 x^2 + 9/4 x - 2$ we type it in the *Input field*, not in the form as it is, but in the general form $f(x) = a x^2 + b x + c$. We have not to worry about this because the coefficients of the function are defined by their respective sliders. (The *Input field* is the text box with the label *Input* located at the bottom of GeoGebra window). Press the **ENTER** key on the keyboard and the graph of the given function will appear in Geometry window and the respective equation in Algebra window.

7. Construct the intersection points of the graph with x -axis by selecting the *Intersection* tool for two objects in the respective tool box and clicking one at a time on the graph and x -axis. The points A and B will appear. They help in defining the left and right bounds of the region in study which are the first coordinates of these two points shown in Algebra window.

8. Calculate the area of the enclosed region by executing the command of *Integral*. This is done by typing in the *Input field*: **integral[f(x),l,r]** and pressing the **Enter** key. The calculation of area is needed to make visible the enclosed region under study. Within the enclosed region and in Algebra window is displayed the equation **d = 14.29** corresponding to the area of the colored region. Using the object *Properties* in the dialog box of the equation $d = 14.29$ (click on that equation in Algebra window or on the respective region and it will be displayed) we can change the color of the region by selecting one in the panel of the colors.

9. Calculate the coordinates of the centroid by typing in the *Input bar* the respective expressions for the coordinates: $1/d \int_l^r x f(x) dx$ (don't forget to press the **Enter** key). In Algebra window will be shown the equation **e = 4.5**, the first coordinate of centroid. Type also, $1/d \int_l^r 0.5 f(x)^2 dx$ and press the **Enter** key. In Algebra window will be shown the equation **g = 1.23**, the second coordinate of the centroid. Type $C = (e,g)$, press the **Enter** key and the point C will be displayed in the two windows. That is the centroid point of the enclosed plane region. **C = (4.5, 1.23)**.

The second stage of the procedure: creating the new tool to calculate the centroid of the enclosed region

1. Click on **Tools** in menu bar and in its respective box select **Create New Tool**. In the displayed dialog box and in the window of *Output objects*, under *Select objects in construction* or choose from the

list, open the box by clicking on the small triangle at the right side of the bar. The box will be open and the list of the function, numbers and points will be displayed:

- Functionf: Integral[f(x),l,r]
- Number d: integral[f(x),l,r]
- Number e: 1/d integral[x f(x),l,r]
- Number g: 1/d integral[0.5 f(x)²,l,r]
- Point A: Intersection point of f, xAxis
- Point B: Intersection point of f, xAxis
- Point C: (e,g)

Successively select each one of the elements of the list by using the pointer of the mouse or the pointer of the finger pad and pressing the Enter key or hitting the finger pad. So, place the pointer on the Functionf: integral[f(x),l,r] and press the Enter key. Do this operation for all the elements of the list and they will be transferred to the box of the Output objects. Each time an element is transferred the box of the list must be reopened until there is no one left in it.

2. Press Next button and the box of the Input objects will be open. It contains the numbers l, r, a, b, c. This box is opened to be assured that these numbers are there in. We have nothing else to do here.

3. Press Next button and the box of Name&Icon will be opened. In the box of Tool Name we type the name of the new tool, for instance Centroid (Pol.2) related to the enclosed regions bounded by the curves of the polynomials of grade two. In the Tool help text, type **Select the slider of r and successively type the numbers l, a, b and c in the table displayed.** In the box of the Command Name type Centroid. Pressing the Icon button we can upload an icon found from other picture sources as we desire. Press Finish and then OK. The centroid tool is successfully created.

4. Now, delete all the objects under Dependent objects of Algebra window. All that was constructed disappears. The deletion is done in order we understand that the algorithm works with any functions of second degree. We can choose the function by changing the values of the coefficients of polynomial and the minimum and maximum of the slider if needed.

The third stage of the procedure: Calculation of centroid. Suppose we want to calculate the centroid of enclosed plane region bounded by the graph of the function $f(x) = -1/4 x^2 + 2x$ and x-axis. To calculate it, we must be in the GeoGebra window in which the centroid tool for the functions of grade two is created and the respective sliders are displayed in the Geometry window.

1. Firstly, we type in the Input field $f(x) = -1/4 x^2 + 2x$, press the Enter key and the graph is displayed. Selecting the Intersection tool for two objects in the respective tool box and clicking one at a time on the graph and x-axis we construct the intersection points A = (0,0) and B = (8,0). These are needed to define the values of l and r.

2. In the sliders we set the values: a = -0,25, b = 2, c = 0, l = 0 and r = 8. For purpose of accuracy it is good the sliders increment be set 0.01. So, to set the value - 0.25, we can move the point (the slider) using the mouse or the finger pad to bring it close to this value, but it is difficult to position it at - 0.25. So, we click on the point and then use the Arrow buttons of the keyboard to move the point left or right until is reached the exact value of - 0.25.

3. Select Centroid tool and immediately click, firstly, on the slider r (representing the right bound of the region). After the click is displayed a table. Perform the following operations in a successive way: type l and press the Enter key in the keyboard after it, type a and press the Enter key in the keyboard after it, type b and press the Enter key in the keyboard after it, type c and press the Enter key in the keyboard after it. After the last command it is displayed the graph in the Geometry window and the centroid, the position of centroid C in Geometry window and its coordinates in Algebra window: C = (4, 1.6). To make visible the enclosed region we calculate its area by typing in the Input field **integral[f(x),l,r]**.

Note: It is very important that to get the centroid after the selection of the Centroid tool, we firstly click on the slider r, then we successively perform the other operations in the table displayed in this order: type l and press Enter, type a then press Enter, type b then press Enter, type c then press Enter. Applying any other order will make us fail. Clicking firstly on the slider r means that the value of r is registered in the table. The third stage serves also as a test that our procedure for the calculation of the centroid is a correct one. On the other hand, it shows that the calculation of the centroid needs fewer steps and the time is very short.

If have an applet access we can calculate the centroid of such type of enclosed plane figure resetting the applet at any time by clicking the sliders usually in the top right corner. By moving the sliders at new values we investigate what happens with the centroid when moving the 'slider' left and right.

Centroid of a plane shape bounded by a polynomial curve of fourth degree and the horizontal axis. The centroid tool created for the above example is valid for the curves of second degree only. We have to create new centroid tools for the curves of higher degrees: a new tool for the third degree curves, a new tool for the fourth degree curves and so on. This is done by constructing additional sliders corresponding to the additional coefficients of the polynomial and creating the new tool from the beginning. Also, we have to create new tools for exponential curves, logarithmic curves, trigonometric curves and other types of curves. We note here that when we have to calculate the centroid of an enclosed plane region bounded by curves of different types then, all the sliders needed for each one of them must be present in Geometry window. We have no time and no space in this paper to deal with many curves of different types. We will be focused on the enclosed plane regions bounded by polynomial functions up to fourth degree. The centroid tool created for the polynomial of fourth degree is valid for the polynomials of lower degrees as well. This is achieved by taking the coefficients of monomials of higher degrees equal to zero (put the sliders corresponding to such coefficients to zero). After creating the centroid tool valid for such example one can understand that it is not difficult to create centroid tool for the cases of polynomials of higher degrees (just construct additional sliders and create the new tool from the beginning).

Example of fourth degree curve. Consider the case of the enclosed plane region bounded by the polynomial function $f(x) = -0.2x^4 + 1.5x^3 - 3x^2 + 2x + 6$, x-axis and the part on the right side of y-axis (Fig.2).

The first stage of the procedure: creating the sliders and graphing the function. Knowing in details the steps of the procedure applied in the first example we present here fewer steps and keep them shorter. Some of the steps are skipped.

1. Create the sliders for the polynomial coefficients and for the left bound and the right bound of the enclosed plane region, as above. The general form of the function of fourth degree is $f(x) = ax^4 + bx^3 + cx^2 + dx + e$. There are five coefficients, hence we construct five sliders, one for each one of them, and two more sliders corresponding to the left bound and the right bound. Name them after the coefficients and the bounds and choose the maximum and minimum as desired and containing the coefficients of the above given polynomial, while is recommended that the increment be 0.01.

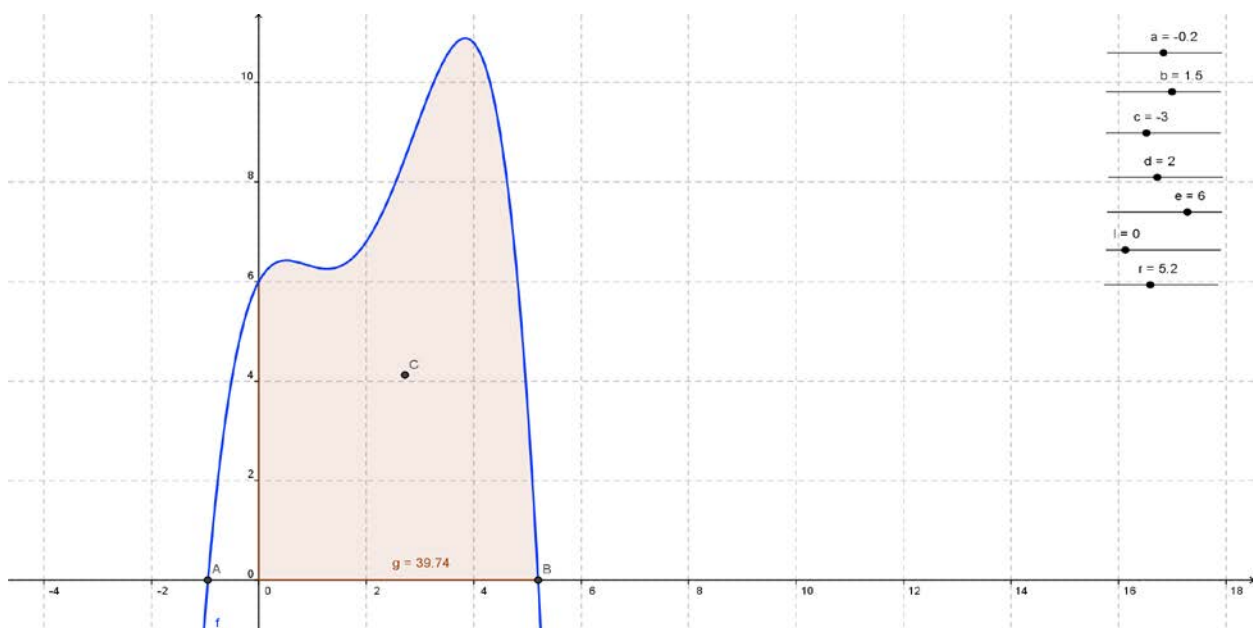


Fig. 2

2. Enter in the Input field the function $f(x) = -0.2x^4 + 1.5x^3 - 3x^2 + 2x + 6$ and press Enter.

3. Construct the intersection points of the curve with x-axis by using the Intersection for two objects tool. We can see that the intersection points are $A = (-0.95, 0)$ and $B = (5.2, 0)$, (look at Fig.2). The steps 2 and 3 are performed to see the position of the curve and to define the coordinates of the intersection points. The errors done are of 1/100 rank.

4. Arrange the positions of the sliders, by moving the sliders (don't forget to select the Move button) at the values: $a = -0.2$, $b = 1.5$, $c = -5$, $d = 2$, $e = 6$, $l = 0$ and $r = 5.2$. These are the values of the parameters of the fourth grade function and of the left and the right bound of the region (the left bound is 0).

5. Graph the function $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ by typing it in the Input field. The coefficients of the function are defined by the values in their respective sliders. Press the **ENTER** key on the keyboard and the graph of the given function will appear in Geometry window (it is the graph constructed earlier) and the respective equation in Algebra window.

6. Calculate the area of the enclosed region by executing the command of integral. This is done by typing in the Input field: $\text{integral}[f(x), l, r]$ and pressing the Enter key. The calculation of area is needed to make visible the enclosed region under study. Within the enclosed region and in Algebra window is displayed the equation $g = 39.74$ corresponding to the area of the colored region. Using the Object Properties in the dialog box of the equation $g = 39.74$ (click on that equation in Algebra window or in the respective region and it will be displayed) we can change the color of the region by selecting one in the panel of the colors.

7. Calculate the coordinates of the centroid by typing in the Input field the respective expressions for the coordinates: $1/g \text{ integral}[xf(x), l, r]$ (don't forget to press the Enter key). In Algebra window will be shown the equation $h = 2.72$, the first coordinate of centroid. Type also, $1/g \text{ integral}[0.5f(x)^2, l, r]$ and press the Enter key. In Algebra window will be shown the equation $j = 4.12$, the second coordinate of centroid. Type $C = (h, j)$, press the Enter key and the point C will be displayed in the two windows. That is the centroid point of the enclosed plane region. $C = (2.72, 4.12)$.

The second stage of the procedure: creating the new tool to calculate the centroid of the enclosed region.

1. Click on Tools in menu bar and in its respective box select Create New Tool. In the dialog box displayed and in the window of Output objects, under Select objects in construction or choose from the list, open the box by clicking on the small triangle at the right side of the bar. The box will be opened and the list of the function, numbers and points will be displayed (as in the first example). Successively select each one of the elements of the list by using the pointer of the mouse or the pointer of the finger pad and then press the Enter key or hit the finger pad. Do this operation for all the elements of the list and they will be transferred to the box of the Output objects. Each time an element is transferred the box of the list must be reopened until there is no one left in it.

2. Press Next button and the box of the Input objects will be opened. It contains the numbers l, r, a, b, c, d, e . This box is opened to be assured that these numbers are there in.

3. Press Next button and the box of Name&Icon will be opened. In the box of Tool Name we type the name of the new tool, for instance Centroid (Pol.4) related to the enclosed regions bounded by the curves of the polynomials of grade four. In the Tool help text, type **Select the slider of r and successively type the numbers l, a, b, c, d and e in the table displayed**. In the box of the Command Name type Centroid. Pressing the Icon button we can upload an icon found from other picture sources as we desire. Press Finish and then OK. The centroid tool is successfully created.

4. Now, delete all the objects under Dependent objects of Algebra window. All that was constructed disappears. The deletion is done in order we understand that the algorithm works with any functions of fourth degree. We can choose the function by changing the values of the coefficients of polynomial and the minimum and maximum of the slider if needed.

The third stage of the procedure (the test of the created tool). Suppose we want to calculate the centroid of the enclosed plane region bounded by the graph of the above function $f(x) = -0.2x^4 + 1.5x^3 - 3x^2 + 2x + 6$ and x-axis. To calculate it, we must be in the GeoGebra window in which the centroid tool for the functions of grade four is created and the respective sliders are displayed in the Geometry window.

1. Firstly, we check the sliders if their values are $a = -0.2$, $b = 1.5$, $c = -5$, $d = 2$, $e = 6$, $l = 0$ and $r = 5.2$. If not, we have to put these values on them.

2. Select Centroid tool and immediately click, firstly, on the slider r (representing the right bound of the region). After the click is displayed a table. Perform the following operations in a successive way: type l and press the Enter key in the keyboard after it, type a and press the Enter key in the keyboard after it, type b and press the Enter key in the keyboard after it, type c and press the Enter key in the keyboard after it, type d and press the Enter key in the keyboard after it, type e and press the Enter key in the keyboard after it. At the end of the last command is displayed the graph in the Geometry window and the centroid, the position of centroid C in Geometry window and its coordinates in Algebra window: $C = (2.72, 4.12)$. To make visible the enclosed region we calculate its area by typing in the Input field $\text{integral}[f(x),l,r]$.

Centroid of a plane shape bounded by two polynomial curves of at most fourth degree. In order to show that the tool of centroid created above is valid for any polynomial function of at most fourth degree, in this case, we consider the enclosed plane region bounded by two curves: one corresponding to the polynomial function of fourth degree and the other to the polynomial function of third degree. Suppose we have the enclosed plane region bounded by $f_1(x) = -0.2x^4 + 1.5x^3 - 3x^2 + 2x + 6$ and $f_2(x) = -0.1x^3 + 0.6x^2 + 0.4x + 2.4$ (look their graphs at Fig.3, the region is the one with light rose colour).

The first stage of the procedure: creating the sliders and graphing the functions. In this case we need to create a new centroid tool from the beginning because we have two functions. We follow the same steps as in the case 2.

1. Create the sliders for the coefficients of the two polynomial functions of the fourth degree: $f_1(x) = ax^4 + bx^3 + cx^2 + dx + e$, $f_2(x) = a'x^3 + b'x^2 + c'x + d'$, and for the left bound and the right bound of the enclosed plane region, as above. There are ten coefficients, hence we construct ten sliders, one for each one of them, and two more sliders corresponding to the left bound and the right bound (twelve sliders in all). Name them after the coefficients and the bounds and choose the maximum and minimum as desired and containing the coefficients of the above given polynomials, while is recommended that the increment be 0.01.

2. Enter in th Input field the functions $f_1(x) = -0.2x^4 + 1.5x^3 - 3x^2 + 2x + 6$ and $f_2(x) = -0.1x^3 + 0.6x^2 + 0.4x + 2.4$ to graph them.

3. Construct the intersection points of the two curves by using the Intersection for two objects tool and clicking on the two graphs. The intersection points are $A = (-0.71, 2.46)$ and $B = (4.71, 7.14)$, (look at Fig.3). The steps 2 and 3 are performed to see the positions of the curves and to define the coordinates of the intersection points.

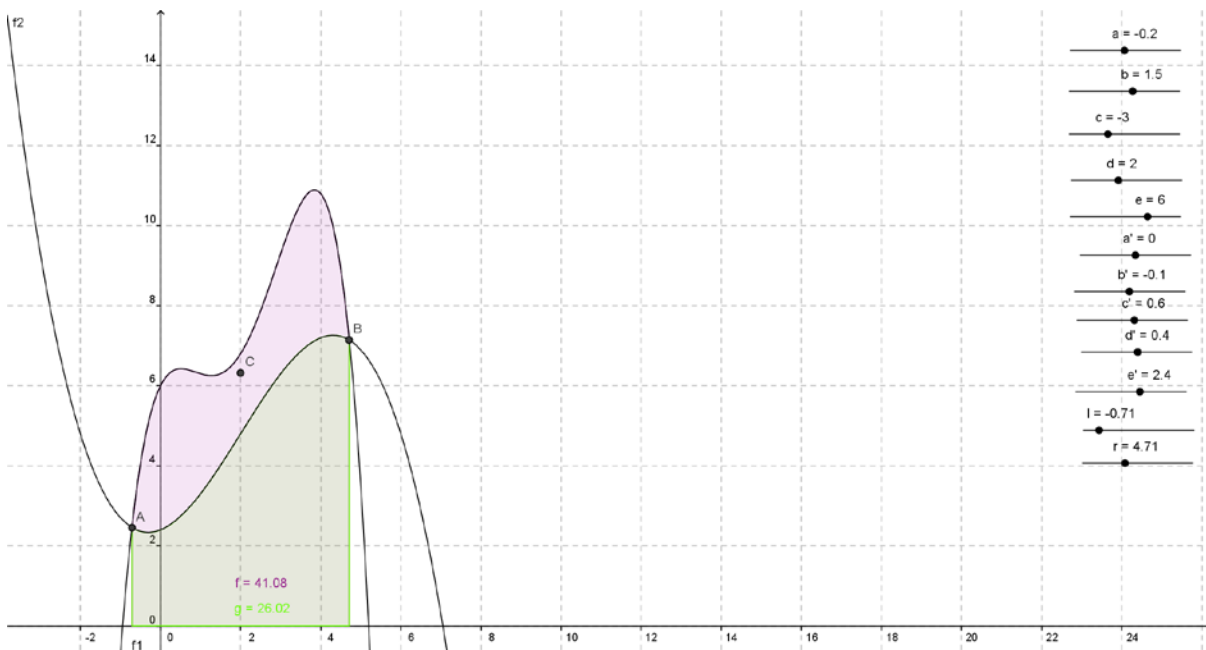


Fig. 3.

4. Arrange the positions of the sliders, by moving the sliders (don't forget to select the Move button) at the values: $a = -0.2$, $b = 1.5$, $c = -3$, $d = 2$, $e = 6$, $a' = 0$, $b' = -0.1$, $c' = 0.6$, $d' = 0.4$, $e' = 2$, $l = -0.71$ and $r = 4.71$. These are the values of the parameters of the above functions and of the left and the right bound of the plane region.

5. Graph the functions $f_1(x) = a x^4 + b x^3 + c x^2 + d x + e$ and $f_2(x) = a' x^4 + b' x^3 + c' x^2 + d' x + e'$ by successively typing them in the Input field. The coefficients of the functions are defined by the values in their respective sliders. The graphs of the given functions will appear in Geometry window and their respective equations in Algebra window.

6. Calculate the areas of the enclosed regions bounded by each graph and the x-axis, respectively. This is done by executing the command of integral. Type in the Input field: **integral[f₁(x),l,r]** and press the Enter key, type again in the Input field: **integral[f₂(x),l,r]** and press the Enter key. The calculation of areas is needed to make visible the enclosed region under study. Within the enclosed regions and in Algebra window are displayed the equations **f = 41.08** and **g = 26.02** corresponding to the areas of the different colored regions. Using the Object Properties in the dialog box of the equation **f = 41.08** and **g = 26.02** (click on each equation in Algebra window or in their respective regions and it will be displayed) we change the colors of the regions by selecting one in the panel of the colors in order to contrast them and make more visible the region for which we want to calculate the centroid.

7. Calculate the area of the enclosed region bounded by the two graphs. This is done by executing the command of integral: **integral[f₁(x) - f₂(x),l,r]**. In Algebra window will be displayed the equation **h = 15.07**. Also, will appear a region corresponding to the area bounded by the graph of the function **f₁(x) - f₂(x)** and x-axis, but to get rid of the confusion, make it invisible by clicking on the bullet corresponding to h in Algebra window.

8. Calculate the coordinates of the centroid by typing in the Input bar the respective expressions for the coordinates: $1/h \text{ integral}[x(f_1(x) - f_2(x)),l,r]$ (don't forget to press the Enter key). In Algebra window will be shown the equation **j = 2**. This is the first coordinate of centroid. Type also, $1/h \text{ integral}[0.5[(f_1(x))^2 - (f_2(x))^2],l,r]$ and press the Enter key. In Algebra window will be shown the equation **k = 6.31**, the second coordinate of centroid. Type **C = (j, k)**, press the Enter key and the point C will be displayed in the two windows. That is the centroid point of the enclosed plane region, **C = (2, 6.31)**.

The second stage of the procedure: creating the new tool to calculate the centroid of the enclosed region between the two curves

1. Click on Tools in menu bar and in its respective box select Create New Tool. In the dialog box displayed and in the window of Output objects, under Select objects in construction or choose from the list, open the box by clicking on the small triangle at the right side of the bar. The box will be opened and the list of the functions, numbers and points will be displayed as in the previous examples. Successively select each one of the elements of the list by using the pointer of the mouse or the pointer of the finger pad and then press the Enter key or hit the finger pad. Do this operation for all the elements of the list and they will be transferred to the box of the Output objects. Each time an element is transferred the box of the list must be reopened until there is no one left in it.

2. Press Next button and the box of the Input objects will be opened. It contains the numbers l, r, a, b, c, d, e, a', b', c', d', e'. This box is opened to be assured that these numbers are there in.

3. Press Next button and the box of Name&Icon will be opened. In the box of Tool Name we type the name of the new tool, for instance Centroid (Pol-Between) related to the enclosed regions bounded by the curves of the two polynomials of grade at most four. In the Tool help text, type **Select the slider of r and successively type the numbers l, a, b, c, d, e, a', b', c', d', e' in the table displayed**. In the box of the Command Name type Centroid. Pressing the Icon button we can upload an icon found from other picture sources as we desire. Press Finish and then OK. The centroid tool is successfully created.

4. Now, delete all the objects under Dependent objects of Algebra window. All that was constructed disappears. The deletion is done in order we understand that the algorithm works with any functions of at most fourth degree. We can choose the function by changing the values of the coefficients of polynomials and the minimum and maximum of the slider if needed.

The third stage of the procedure (the test of the created tool). Suppose we want to calculate the centroid of the enclosed plane region bounded by the graphs of the above functions $f_1(x) = -0.2 x^4 + 1.5 x^3 - 3 x^2 + 2 x + 6$ and $f_2(x) = -0.1 x^3 + 0.6 x^2 + 0.4 x + 2$. To calculate it, we must be in the GeoGebra window in which the centroid tool for the functions of grade four is created and the respective sliders are displayed in the Geometry window.

1. Firstly, we check the sliders if their values are: $a = -0.2$, $b = 1.5$, $c = -3$, $d = 2$, $e = 6$, $a' = 0$, $b' = -0.1$, $c' = 0.6$, $d' = 0.4$, $e' = 2$, $l = -0.71$ and $r = 4.71$. If not, we have to put these values on them.

2. Select Centroid tool and immediately click, firstly, on the slider r (representing the right bound of the region). After the click a table is displayed. Perform the following operations in a successive way: type l and press the Enter key in the keyboard after it, type a and press the Enter key, type b and press the Enter key, type c and press the Enter key, type d and press the Enter key, type e and press the Enter key, type a' and press the Enter key, type b' and press the Enter key, type c' and press the Enter key, type d' and press the Enter key, type e' and press the Enter key. After the last command we see the graphs displayed in the Geometry window and the centroid, the position of centroid C in Geometry window and its coordinates in Algebra window: $C = (2, 6.31)$. To make visible the enclosed region we calculate areas corresponding to each curve by typing in the Input field: **integral[f₁(x),l,r]** and press the Enter key, type again in the Input field: **integral[f₂(x),l,r]** and press the Enter key. Using the object Properties in the dialog box of the equation $f = 41.08$ and $g = 26.02$ (click on each equation in Algebra window or in their respective regions and it will be displayed) we change the colors of the regions by selecting one in the panel of the colors in order to contrast them and make more visible the region between the curves and for which we want to calculate the centroid.

CONCLUSIONS. The new tool created to calculate the centroid of the enclosed plane regions bounded by curves of a special class of functions is very useful and of practical value. If the tool is created for a class of functions then it is possible to calculate the centroid of any plane region bounded by curves of any function belonging to that class. The time needed to calculate the centroid is un-compared with the time when we use the traditional way for calculus. Also, it is possible to calculate the centroid of a thin plate of metal or plastic, having the shape of the plane regions treated in this paper, by scanning it and exporting to GeoGebra window. The position of the centroid point in the plate is more accurate than the position found by using other practical ways. The advantages of using GeoGebra in calculating the centroid are the time and the accuracy.

Knowing the above-described procedure of how is created the new tool, anyone, having skills in GeoGebra, can create new tools to calculate the centroid of the enclosed plane region bounded by polynomial curves of higher degree than four, exponential curves, logarithmic curves, trigonometric curves and other types of curves. We note here that when we have to calculate the centroid of an enclosed plane region bounded by curves of different types then, all the sliders needed for each one of them must be present in Geometry window.

The results so far obtained using GeoGebra show that a mathematic work based on modeling and simulation of phenomenon allow to develop a series of skills required in today's world, especially in the research work. The power of modeling through continuous regulation also generates a significant change in the concepts of mathematics and physics in all aspects.

By exploring with GeoGebra the students, teachers and GeoGebra users in general gain a great deal of insight about how work gets done in the "real world", they gain personal and professional qualities and experiences that contribute to further career success.

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Geogebra: вычисление центра тяжести

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Аннотация. Наша работа является результатом исследования, проведенного в определенном направлении для решения проблем физики, используя программу Geobra: вычисление центра тяжести. Используя Geogebra, можно выполнять моделирование физических феноменов из курса механики и решать вычислительные задачи. GeoGebra предлагает множество команд и одной из команд является «центр тяжести», определяющая координаты центра тяжести многоугольника, но мы создали новый инструмент для вычисления координат центра тяжести плоского района, ограниченного кривыми. Наша работа является частью работы многих пользователей GeoGebra, результатом которой явился богатый фонд виртуальных инструментов, примеров и опытов GeoGebra, которые станут доступными многим учителям и практикам во всем мире.

Ключевые слова: центр тяжести; инструментарий центра тяжести; возможности GeoGebra.