Estimation of Small Rainfall Events Impact on the Urban Runoff by Analytical Model

Ali Salajegheh
Elham Forootan
Mohammad Mahdavi
Hassan Ahmadi
Forood Sharifi

Tehran University, Iran
Professor
E-mail: salajegh@ut.ac.ir
Islamic Azad University, Iran
Student
E-mail: eliforootan@yahoo.com
Tehran University, Iran
Professor
Islamic Azad University, Iran
Professor
Soil and Water Conservation Research Center, Iran
Assistant professor

Abstract. The amount of impervious area in urban catchment increases because of developed conditions, resulting in the increase of peak discharge rate and runoff volume. The estimation of runoff quantity helps to design stormwater control facility. This research considers analytical model for urban runoff estimation in the test catchment located in semiarid region based on efficient impervious area. To study the significance of small rainfall events, the rainfall events less than 10 mm was considered in the analytical model. The results of this study revealed that the omission of rainfall volume less than 6 mm does not change the maximum relative error value remarkably and the analytical model is not sensitive to the rainfall events less than 6 mm. This omission will simplify the statistic calculations for providing rainfall characteristics parameters of analytical model.

Keywords: Analytical model; Probability density function; Simulation model.

Introduction
In a rural catchment the majority of land surface is pervious and covered by vegetation, therefore; canopy interception, soil infiltration and evapotranspiration tend to be large; while direct surface runoff tends to be small. In urban catchment because of developed conditions, the amount of impervious area increases which tends to increase peak discharge rate and runoff volume and decrease the infiltration volume and the interflow and baseflow in the downstream water course. More frequent occurrence of large discharges may intensify channel erosion problems which cause degradation of water quality. In order to decrease flood damages and environmental impacts, storm water control facilities are necessary. The urbanized watersheds consist of open drains to complex sewer networks, as a result simulation of this response system for urban planning and development is very difficult. The estimation of runoff quantity helps to design of stormwater control facility in the proper way. There are some methods and approaches to estimate urban runoff quantity, because of the complex physical mechanism of rainfall-runoff transformation.

The continuous simulation approach involves conceptual modeling of the physical system recognizing not only the properties of storms but also the accumulative effect of closely-spaced storms (Loganathan and Delleur, 1984). The statistics from continuous simulation model output
are usually believed to be more reliable compared to other types of models. Derived probability theory was first outlined by Benjamin and Cornell (1970) and the application of derived probability distribution theory to water resources engineering and hydrological science was pioneered by Eagleson (1972, 1978). Many studies have been performed in this field, for example; the results of proposed analytical model for urban runoff estimation (Guo, 1998) revealed its accuracy in the urban catchment of Toronto. Moreover, two analytical models (Chen and Adams, 2007) were developed for runoff estimation which results showed that the analytical model based on the infiltration parameter was better than the analytical model based on runoff coefficient. The "effective" impervious area, EIA, defined as the portion of "mapped" impervious area, MIA, (Sutherland, 1995), one of the most important parameters for rainfall-runoff model of urban catchment has not been considered in the previous analytical models. Moreover, the effect of small rainfall events (Smith and Schreiber, 1974) in analytical models has not been investigated. In this paper analytical model, based on effective impervious area and the significance of small rainfall events (less than 10 mm) for runoff estimation has been considered in the test catchment in semiarid region.

**Methods**

Chen and Adams (2007) formulated rainfall-runoff transformation with the consideration of Horton infiltration in order to enhance the physics of analytical models. Due to the explicit consideration of the infiltration process, rainfall losses can be more accurate. However, major inputs of this model are described by probability distributions of the rainfall event characteristics, namely, rainfall event volume (v), duration (t) and interevent time (b). (table 1)

The formula, suggested by Chen and Adams (2007) assumed total impervious area, connected to drainage system of urbanize catchment whereas in the developed analytical model, the fraction of impervious area, directly connected to drainage system ($\beta$) is considered for precise estimation of runoff and is calculated as following:

\[
v_r = \begin{cases} 
0 & v \leq s_{di} \\
\beta(v - s_{di}) & s_{di} \leq v \leq s_{dp} + s_{iw} + \frac{f_c}{k} \\
\beta(v - s_{di}) + \alpha(v - s_{dp} - s_{iw} - \frac{f_c}{k}) & v > s_{dp} + s_{iw} + \frac{f_c}{k} 
\end{cases}
\]

$\alpha$ = The fraction of impervious area

$s_{di}$ = Impervious area depression storage in mm

$s_{dp}$ = Pervious area depression storage in mm

$v$ = Rainfall event volume in mm

$f_c$ = Ultimate infiltration capacity in the Horton equation in mm/h

$s_{iw}$ = Initial soil wetting infiltration loss in the pervious area of the catchment in mm

The Probability Density Function (PDF) of runoff volume is expressed as:

\[
f_{v_r}(v_r) = \begin{cases} 
1 - e^{-s_{di} / \zeta} & v_r = 0 \\
\frac{\zeta}{\zeta} e^{-\frac{v_r}{\zeta} + s_{di}} & 0 < v_r \leq h(s_{dp} + s_{iw} + s_{di} + \frac{f_c}{k}) \\
\frac{\zeta}{\beta} e^{-\frac{v_r}{\beta} + s_{di}} & v_r > h(s_{dp} + s_{iw} - s_{di} + \frac{f_c}{k}) 
\end{cases}
\]
Table 1: Rainfall characteristics parameter (Chen and Adams, 2007)

<table>
<thead>
<tr>
<th>Applicable range</th>
<th>Parameter</th>
<th>Exponential PDF</th>
<th>Rainfall characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq v &lt; \infty$</td>
<td>$\zeta = \frac{1}{v}$</td>
<td>$f_v(v) = \zeta e^{-\zeta v}$</td>
<td>Volume, $v$ (mm)</td>
</tr>
<tr>
<td>$0 \leq t \leq \infty$</td>
<td>$\lambda = \frac{1}{t}$</td>
<td>$f_t(t) = \lambda e^{-\lambda t}$</td>
<td>Duration, $t$ (h)</td>
</tr>
<tr>
<td>$0 \leq b \leq \infty$</td>
<td>$\psi = \frac{1}{b}$</td>
<td>$f_b = \psi e^{-\psi b}$</td>
<td>Interevent time, $b$ (h)</td>
</tr>
</tbody>
</table>

Study Area
The test catchment with an area of 67.8 ha, selected for this study is located in the upper part of District No.22 of Tehran city, Iran. The catchment is composed of residential land and green area. The physical characteristics of test catchment used in the two models are given in table 2.

There is no rainfall gauge in the test catchment, therefore long term rainfall series is taken from Mehrabad rainfall station and the data available from 1959 to 1998 is used for performing this study. For analytical model, the long term rainfall record of the Mehrabad station is analyzed and probability density function of rainfall event volume, duration, interevent time is calculated for estimation of three statistical parameters ($\zeta, \lambda, \psi$) on the basis of 1h interevent time definition.

Table 2: physical characteristics of the test catchment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Catchment area (ha)</td>
<td>67.8</td>
<td>Impervious area depression storage (mm)</td>
<td>1.25</td>
</tr>
<tr>
<td>Slope (%)</td>
<td>5.85</td>
<td>Pervious area depression storage (mm)</td>
<td>2.5</td>
</tr>
<tr>
<td>n of Manning of impervious area</td>
<td>0.011</td>
<td>Initial infiltration capacity (mm/h)</td>
<td>36</td>
</tr>
<tr>
<td>n of Manning for pervious area</td>
<td>0.15</td>
<td>Ultimate infiltration capacity (mm/h)</td>
<td>7.2</td>
</tr>
<tr>
<td>Catchment width (m)</td>
<td>276</td>
<td>Infiltration decay coefficient (1/h)</td>
<td>4.5</td>
</tr>
</tbody>
</table>

To investigate the significance of small rainfall events for runoff estimation in analytical model, three statistical parameters of analytical models are estimated by omitting rainfall volume less than 1,2,....10, respectively. Moreover; The contribution of small rainfall events for runoff generation and the proportion of total rainfall are given in table 3.

Table 3: The characteristics of small rainfall events in the test catchment

<table>
<thead>
<tr>
<th>Omitted Rainfall events</th>
<th>Small rainfall event volume to total volume (%)</th>
<th>Small rainfall event to total event (%)</th>
<th>Contribution for Runoff production (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rainfall event less than 1 mm</td>
<td>0.8</td>
<td>33</td>
<td>0</td>
</tr>
<tr>
<td>Rainfall event less than 2 mm</td>
<td>1.95</td>
<td>41</td>
<td>0.18</td>
</tr>
<tr>
<td>Rainfall event less than 3 mm</td>
<td>3.32</td>
<td>52</td>
<td>0.82</td>
</tr>
<tr>
<td>Rainfall event less than 4 mm</td>
<td>4.7</td>
<td>56</td>
<td>1.65</td>
</tr>
<tr>
<td>Rainfall event less than 5 mm</td>
<td>6.27</td>
<td>60</td>
<td>2.7</td>
</tr>
<tr>
<td>Rainfall event less than 6 mm</td>
<td>7.9</td>
<td>63</td>
<td>3.92</td>
</tr>
<tr>
<td>Rainfall event less than 7 mm</td>
<td>12</td>
<td>70</td>
<td>7</td>
</tr>
<tr>
<td>Rainfall event less than 8 mm</td>
<td>12.6</td>
<td>71</td>
<td>7.5</td>
</tr>
<tr>
<td>Rainfall event less than 9 mm</td>
<td>15.6</td>
<td>75</td>
<td>10</td>
</tr>
<tr>
<td>Rainfall event less than 10 mm</td>
<td>18.1</td>
<td>79</td>
<td>11.1</td>
</tr>
</tbody>
</table>
Table 4: Maximum absolute relative error values

<table>
<thead>
<tr>
<th>Omitted Rainfall events</th>
<th>Maximum absolute relative error</th>
<th>Omitted Rainfall events</th>
<th>Maximum absolute relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Events less than 1 mm</td>
<td>0.11</td>
<td>Events less than 1 mm</td>
<td>0.16</td>
</tr>
<tr>
<td>Events less than 2 mm</td>
<td>0.13</td>
<td>Events less than 2 mm</td>
<td>0.19</td>
</tr>
<tr>
<td>Events less than 3 mm</td>
<td>0.14</td>
<td>Events less than 3 mm</td>
<td>0.2</td>
</tr>
<tr>
<td>Events less than 4 mm</td>
<td>0.15</td>
<td>Events less than 4 mm</td>
<td>0.2</td>
</tr>
<tr>
<td>Events less than 5 mm</td>
<td>0.16</td>
<td>Events less than 5 mm</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Results

As it can be seen in Fig 1, the omission of small rainfall volume two parameters of $\lambda, \zeta$ have descending trend but $\psi$ parameter does not change regularly.

Also, it is found that 33% of rainfall events were less than 1 mm (table 3). Based on the result of this research these events have not generated runoff in the study area whereas comparison of the analytical model runoff with whole events and without 1 mm events resulted in the maximum absolute relative error of 0.109.

The rainfall events less than 2, 3, 4, 5, 6 involve 52%, 56%, 60%, 63% of total events, respectively and the proportion of these volumes to total rainfall volume is 1.95, 3.32, 4.7, 6.27, 7.2 percent. On the basis of analytical model (with all rainfall data), the contribution of these rainfall
volumes to runoff production were 0.18, 0.82, 1.65, 2.7, and 3.92 percent respectively. Moreover; the maximum absolute relative errors by omitting these rainfall volume were 0.132, 0.142, 0.150, 0.158, 0.158; respectively (Fig 2).

With the omission of rainfall event less than 7 mm which consists of 70% of total events, maximum absolute relative error increases to 0.192 of this rainfall. Moreover, 7% of total runoff was produced by these events. Omitting rainfall volumes less than 8,9,10 mm resulted in maximum absolute relative error of 0.20, 0.20, 0.21 respectively, whereas their proportions to total events were 71,75,79. As it can be seen (Fig2), the difference of maximum absolute error increases by omitting rainfall events less than 7 mm which indicates that omitting small rainfall event less than 6 mm does not affect the result of analytical model significantly in this watershed. The rainfall volume less than 8,9 and 10 mm produce 7.5,10 and 11.1 percent of total runoff in this urban catchment. This study indicates that without considering the rainfall less than 6 mm, this analytical model provides well results as well as considering total rainfall in the test catchment and this omission will simplify the statistic calculations for providing rainfall characteristic parameters.

Conclusion
Effective impervious area was considered in rainfall-runoff formulation of analytical model. For investigation of small rainfall event in runoff estimation in analytical model, analysis of long term rainfall series data is performed and it is found that 79 percent of rainfall events have the volume less than 10 mm, therefore the significance of these small rainfall events in analytical model is investigated by the omission of rainfall event volume less than1, 2, 3, 4,5,6,7,..10 mm. The results indicate that the omission of rainfall event less than 6 mm does not change the accuracy of the model results. The contribution of rainfall volume less than 6 mm to runoff generation on the basis of analytical model (all rainfall included) is 3.9 percent while these events involve 7.9 percent of total rainfall volume. This study indicates that 6 mm is the threshold of small rainfall event volume and rainfall event less than 6 mm can be ignored for the statistical calculation for analytical model in the test catchment.

References
Аннотация. Водонепроницаемая площадь городского водосбора увеличивается за счет развития инфраструктуры, что выражается в увеличении уровня пикового расхода воды и объема стоков. Оценка количества сточных вод помогает спроектировать технические сооружения по контролю ливневой воды. В данной работе исследуется аналитическая модель для оценки количества городских сточных вод в тестовом водосборе, расположенном в засушливом районе на основе эффективной водонепроницаемой территории. Для изучения значимости небольшого периода повторяемости ливней, в аналитической модели были взяты за основу осадки менее 10 мм. Результаты данного исследования показали, что уменьшение объема осадков менее 6 мм существенно не меняет максимальную допустимую погрешность, а аналитическая модель не чувствительна к осадкам менее 6 мм. Данное уменьшение упростит статистические расчеты для определения характеристик параметров осадков аналитической модели.

Ключевые слова: аналитическая модель; вероятностная плотность распределения; расчетная модель.